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SUMMARY

start
A boundary-value-collocation procedure was applied to the Williams stress function to determine values of the stress-intensity factor K for ^{49/}single-edge cracks in rectangular-section specimens subjected to three-point bending. The results are presented in terms of the dimensionless quantity $Y^2 = K^2 B^2 W^3 / M^2$ where B and W are the specimen thickness and depth and M is the bending moment at midspan. The values of Y^2 as a function of relative crack depth a/W for three-point bending are appreciably lower than the corresponding values for pure bending (determined previously by the same method) and decrease as the ratio of support span to specimen depth S/W decreases. Plots of Y^2 against a/W are given for values of a/W up to 0.5 and S/W equal to 4 and 8.

The results were relatively insensitive to variations in the spread of the midspan load contact region, which was assumed to be related to the yield strength of the material. The results agreed fairly well with published results derived from experimental compliance measurements; one set gave higher values of Y^2 than the present method, and the other set gave lower values. The plane-strain fracture toughness ^{411/}measurement capacity of three-point bend specimens is somewhat lower than that of four-point bend specimens, but the difference is of negligible practical importance.

end

INTRODUCTION

Various types of crack-notch specimens are used for K_{Ic} plane-strain fracture toughness testing of materials (ref. 1). (Symbols are defined in appendix A.) The value of K_{Ic} determined in such a test is equal to that value of the crack-tip stress-intensity factor K at which the crack becomes unstable and extends abruptly in the opening mode. The stress-intensity factor K is proportional to the applied load and is a function of the

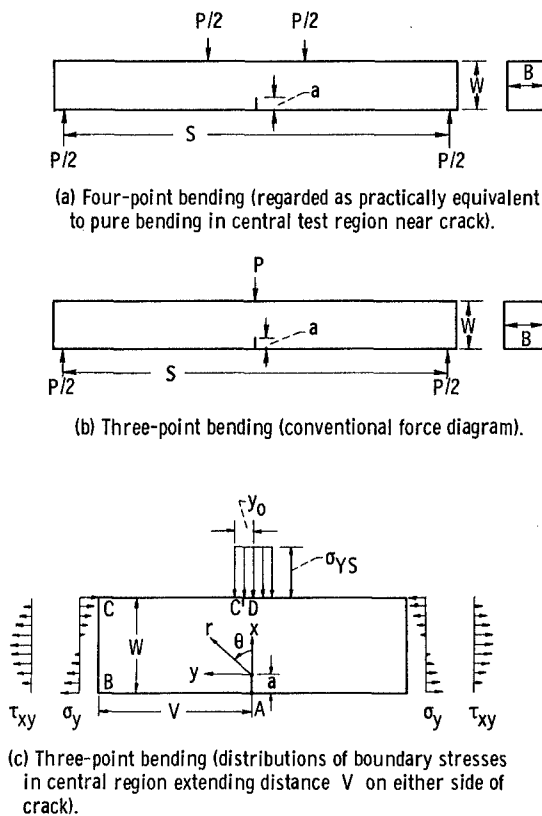


Figure 1. - Diagrammatic representation of beam loading.

specimen dimensions, particularly those of the crack, which depends upon the configuration of the specimen and manner of loading. The determination of an expression for K in terms of these factors for a particular type of specimen will be referred to as the K calibration for that specimen type. Various experimental and analytical methods of stress analysis for K calibration have been developed (ref. 2).

One important class of K_{Ic} specimens, which may be referred to briefly as single-edge-notch specimens, includes those of rectangular cross section having a single crack-notch extending from one edge. The different types of specimen within this class are distinguished primarily by the manner of loading. Previous reports by Gross, Sawley, and Brown dealt with the application of the boundary collocation method of stress analysis for K calibration to single-edge-notch specimens loaded in uniform tension (ref. 3) and to such specimens loaded either in

pure bending or in combined bending and tension (ref. 4).

In practice, single-edge-notch specimens are often tested in three-point bending, whereas the results for pure bending given in reference 4 apply only to ideal four-point bending. In four-point bending (fig. 1(a)), providing that the loads are applied at positions sufficiently distant from the crack, the stress-intensity factor K depends upon four variables only, namely, the applied bending moment M , the crack length a , the specimen depth W , and the thickness B . The K calibration is conveniently expressed in terms of the dimensionless quantity $Y^2 = K^2 B^2 W^3 / M^2$, which is a function of a/W only.

In three-point bending (fig. 1(b)), however, there are two additional independent variables which might affect K . One of these variables is the support span S . For a given value of the maximum bending moment $M = PS/4$, there is a bending moment gradient and a shearing force, both inversely proportional to S . Thus, in terms of the dimensionless form of the K calibration, it is to be expected that Y^2 will be a function of S/W as well as a/W . The second additional variable concerns the distribution of contact pressure around the nominal position of the central loading point. The load is usually applied through a hard, cylindrical roller, and for the present purpose it is assumed that the specimen behaves in the contact region as a rigid-plastic solid with a well-defined yield strength σ_{ys} . The applied load P can then be regarded as evenly

TABLE I. - COMPARISON OF RESULTS OF PRESENT WORK WITH
CORRESPONDING RESULTS FROM REFERENCES 4, 5, 6, AND 7
IN TERMS OF DIMENSIONLESS QUANTITY $K^2 B^2 W^3 / M^2$ AS
FUNCTION OF RELATIVE CRACK LENGTH

Relative crack length, a/W	Results of				
	Boundary collocation	Reference 4	Reference 5	Reference 6	Reference 7
	(a)	(b)	(a)	(c)	
	$Y^2 = K^2 B^2 W^3 / M^2$				
0.10	11.70	12.40	9.44	10.88	10.08
.15	17.30	18.50	13.92	17.28	-----
.20	23.47	25.30	19.20	24.48	22.29
.25	30.78	33.20	26.08	33.12	-----
.30	39.84	42.80	35.20	41.92	-----
.35	51.54	55.20	48.16	57.28	-----
.40	67.18	71.41	63.04	-----	-----
.45	88.78	92.70	83.36	-----	-----
.50	119.74	123.01	108.80	-----	140.48

^aRatio of support span to specimen depth, 8.

^bPure bending.

^cRatio of support span to specimen depth, 10.

distributed over a rectangular area of breadth B (the thickness of the beam) and length $2y_0$ (fig. 1(c)), such that $2y_0 B$ is equal to P/σ_{YS} . Hence, the third variable which affects the value of Y^2 can be taken as $2y_0/W$.

Calibrations for K were computed for S/W values of 4, 6, 8, and 10 to cover what was considered to be a practical range. In the past, a ratio of 8 has been commonly employed, but there is an increasing tendency toward the use of smaller ratios in order to conserve test material. On the other hand, the accuracy of K calibrations for S/W less than 4 was considered dubious because of the increasing difficulty of representing the physical loading conditions accurately as this ratio becomes smaller.

The ratio $2y_0/W$ has an upper limit which is related to the condition for a valid K_{Ic} measurement, namely, that the nominal stress at the crack tip should not exceed the yield strength of the material (ref. 1). From this condition it can be shown that the upper limit of $2y_0/W$ is equal to $2(1 - a/W)^2 W/3S$. Calibrations for K were conducted in parallel by using this upper limit and taking zero to be the lower limit. The differences between the results for the two limits were so small that it was not necessary to consider intermediate values of $2y_0/W$.

Results of experimental compliance measurements on three-point bend specimens have been published by Irwin, Kies, and Smith (ref. 5) and by Kies, Smith, Romine, and Bernstein (ref. 6). Limited results of an analytical study by H. F. Bueckner have been published by Wundt (ref. 7). As shown in table I, there is sufficient lack of agreement between K calibration values derived from the results of these three references to warrant the undertaking of the present study. Furthermore, these references provide no information about the extent to which the K calibration depends on the parameters S/W and $2y_0/W$. In the interest of accurate measurement of K_{IC} , it is important that the extent of the influence of these parameters should be known.

ANALYTICAL AND COMPUTATIONAL PROCEDURE

The method of analysis consists in finding a stress function χ that satisfies the biharmonic equation $\nabla^4 \chi = 0$ and also the boundary conditions at a finite number of stations along the boundary of a single-edge-notched specimen, such as shown in figure 1 (p. 2). The biharmonic equation and the boundary conditions along the crack are satisfied by the Williams stress function (ref. 8). Because of symmetry (fig. 1), the coefficient of the sine terms in the general stress function must be zero; hence,

$$\chi(r, \theta) = \sum_{n=1, 2, \dots}^{\infty} \left\{ (-1)^{n-1} d_{2n-1} r^{n+(1/2)} \left[-\cos\left(n - \frac{3}{2}\right)\theta + \frac{2n-3}{2n+1} \cos\left(n + \frac{1}{2}\right)\theta \right] \right. \\ \left. + (-1)^n d_{2n} r^{n+1} \left[-\cos(n-1)\theta + \cos(n+1)\theta \right] \right\} \quad (1)$$

The stresses in terms of χ obtained by partial differentiation are as follows:

$$\left. \begin{aligned} \sigma_y &= \frac{\partial^2 \chi}{\partial x^2} = \frac{\partial^2 \chi}{\partial r^2} \cos^2 \theta - 2 \frac{\partial^2 \chi}{\partial \theta \partial r} \frac{\sin \theta \cos \theta}{r} + \frac{\partial \chi}{\partial r} \frac{\sin^2 \theta}{r} + 2 \frac{\partial \chi}{\partial \theta} \frac{\sin \theta \cos \theta}{r^2} + \frac{\partial^2 \chi}{\partial \theta^2} \frac{\sin^2 \theta}{r^2} \\ \sigma_x &= \frac{\partial^2 \chi}{\partial y^2} = \frac{\partial^2 \chi}{\partial r^2} \sin^2 \theta + 2 \frac{\partial^2 \chi}{\partial \theta \partial r} \frac{\sin \theta \cos \theta}{r} + \frac{\partial \chi}{\partial r} \frac{\cos^2 \theta}{r} - 2 \frac{\partial \chi}{\partial \theta} \frac{\sin \theta \cos \theta}{r^2} + \frac{\partial^2 \chi}{\partial \theta^2} \frac{\cos^2 \theta}{r^2} \\ -\tau_{xy} &= \frac{\partial^2 \chi}{\partial x \partial y} = \sin \theta \cos \theta \frac{\partial^2 \chi}{\partial r^2} + \frac{\cos 2\theta}{r} \frac{\partial^2 \chi}{\partial r \partial \theta} - \frac{\sin \theta \cos \theta}{r^2} \frac{\partial^2 \chi}{\partial \theta^2} - \frac{\sin \theta \cos \theta}{r} \frac{\partial \chi}{\partial r} - \frac{\cos 2\theta}{r^2} \frac{\partial \chi}{\partial \theta} \end{aligned} \right\} \quad (2)$$

The boundary collocation procedure consists in solving a set of $2m$ simultaneous algebraic equations which correspond to the known values of χ and either $\partial\chi/\partial x$ or $\partial\chi/\partial y$ at m selected stations along the boundary ABCC'D of figure 1(c); thus, values for the first $2m$ coefficients of the Williams stress function are obtained when the remaining terms are neglected. Only the value of the first coefficient d_1 is needed for the present purpose since the stress-intensity factor K is equal to $-\sqrt{2\pi}d_1$, as shown in reference 4.

The required values of χ and its first derivatives at the m selected boundary stations were obtained from distributions of bending moment, shear, and contact stresses (fig. 1(c)), equivalent to the concentrated loads of the conventional force diagram (fig. 1(b)). The equations for these boundary values, in dimensionless form, are as follows:

Along AB

$$\frac{\chi}{P} = 0; \quad W \frac{\partial(\chi/P)}{\partial x} = 0$$

Along BC

$$\frac{\chi}{P} = \frac{6}{BW^3} \left(\frac{S}{2} - V \right) \left[-\frac{x^3}{6} + \frac{x^2}{4} (W - 2a) + \frac{xa}{2} (W - a) + \frac{a^2}{2} \left(\frac{W}{2} - \frac{a}{3} \right) \right]$$

$$W \frac{\partial(\chi/P)}{\partial y} = \frac{-6}{BW^2} \left[-\frac{x^3}{6} + \frac{x^2}{4} (W - 2a) + \frac{xa}{2} (W - a) + \frac{a^2}{2} \left(\frac{W}{2} - \frac{a}{3} \right) \right] \quad (3)$$

Along CC'

$$\frac{\chi}{P} = \frac{S - 2y}{4B}; \quad W \frac{\partial(\chi/P)}{\partial x} = 0$$

Along C'D

$$\frac{\chi}{P} = \frac{1}{4B} \left[S - 2y - \frac{(y - y_0)^2}{y_0} \right]; \quad W \frac{\partial(\chi/P)}{\partial x} = 0$$

The distance V of the end boundary from the crack (fig. 1(c)), was chosen to be approximately $1.5 W$, the exact value being different for different values of a/W as a matter of computational convenience. From physical considerations it is clear that the boundary should be chosen neither close to the crack nor close to the support point, because of the stress-field disturbances near these positions. Preliminary studies established that V/W equal to 1.5 was about optimum and that minor variations from 1.5 had a negligible effect on the K calibration. A more detailed discussion of the effect of the choice of V/W has been given for the case of pure bending in reference 4.

For each set of selected values of the primary variable a/W and the parameters S/W and $2y_0/W$, the collocation computation was carried out four times, using successively 15, 18, 21, and 24 boundary stations. In no case was the variation among the four d_1 values so obtained as great as 1 percent, and in most cases it was a small fraction of 1 percent. From this, together with the nature of the trends of the d_1 values, it was concluded that those values corresponding to 24 boundary stations were very close to the limit values for large numbers of boundary stations. These values were accordingly used to calculate the results reported herein.

The d_1 values so obtained were used to calculate values of the square of the dimensionless stress-intensity coefficient, namely $Y^2 = K^2 B^2 W^3 / M^2$, where $M = PS/4$. In this general form the results are of more immediate utility than they would be in the form of values of K/M for a specimen of specific, arbitrary dimensions.

RESULTS AND DISCUSSION

Dependence of $K^2 B^2 W^3 / M^2$ on S/W and $2y_0/W$

Figure 2 shows plots of Y^2 against S/W for constant a/W values of 0.1, 0.2, and 0.3. In each case the corresponding value of Y^2 for pure bending is shown as a horizontal dashed line for comparison. The values for pure bending were taken from reference 4 and, of course, are independent of S/W . It is clear that the value of Y^2 for three-point loading is always lower than that for pure bending, but that the difference decreases with increasing S/W . For a/W equal to 0.3, the ratio of the value of Y^2 for three-point bending to that for pure bending is 0.87 when S/W is equal to 4 and increases to 0.94 when S/W is equal to 10.

The trend of the curves in figure 2 indicates that Y^2 becomes increasingly sensitive to S/W as this ratio decreases. This is a consequence of the increasing complexity of the overall stress-field pattern with decreasing S/W and is the reason why computations were not conducted for S/W less than 4. It was considered that a K calibration

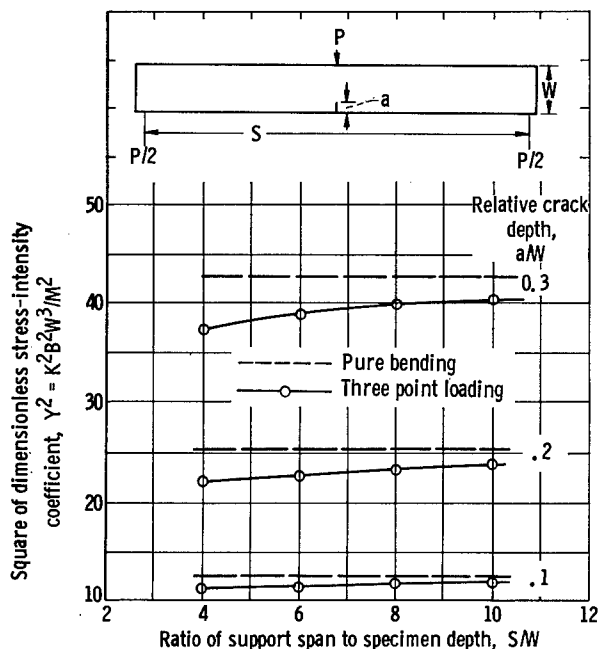


Figure 2. - Dependence of square of stress-intensity coefficient for three-point bending on ratio of support span to specimen depth.

parameter $2y_o/W$ on the K calibration can be considered negligible.

CALIBRATIONS OF K FOR S/W EQUAL TO 4 AND 8

Figure 3 shows plots of the computed values of Y^2 against a/W for S/W equal to 8 and 4, again for the upper limit values of the parameter $2y_o/W$.

The following empirical equations are compact expressions of the same results for values of a/W up to 0.35

$$Y_8^2 = 134 a/W - 247(a/W)^2 + 813(a/W)^3 \quad (4)$$

$$Y_4^2 = 130 a/W - 262(a/W)^2 + 820(a/W)^3 \quad (5)$$

where Y_8^2 refers to S/W equal to 8, and Y_4^2 to S/W equal to 4. These equations were obtained by least-squares-best-fit computer programs for cubics in a/W , incorporating the known condition that K should be zero when a/W is zero. Only the results for a/W up to 0.35 were used in fitting the equations since it is neither desirable nor

for S/W substantially less than 4 would be of very dubious accuracy, in fact, more misleading than useful.

The values of Y^2 plotted in figure 2 are those for the upper limit values of the parameter $2y_o/W$, equal to $2(1 - a/W)^2 W/3S$. If the corresponding values for the lower limit of $2y_o/W$, equal to zero, had also been plotted, they would have been virtually indistinguishable from their companions. For S/W equal to 4, the values of Y^2 for the lower limit of $2y_o/W$ were about 1 percent greater than the plotted values. For S/W equal to 8, the values of Y^2 for the lower limit were less than 1/2 percent greater than the plotted values. For practical purposes, therefore, the effect of the

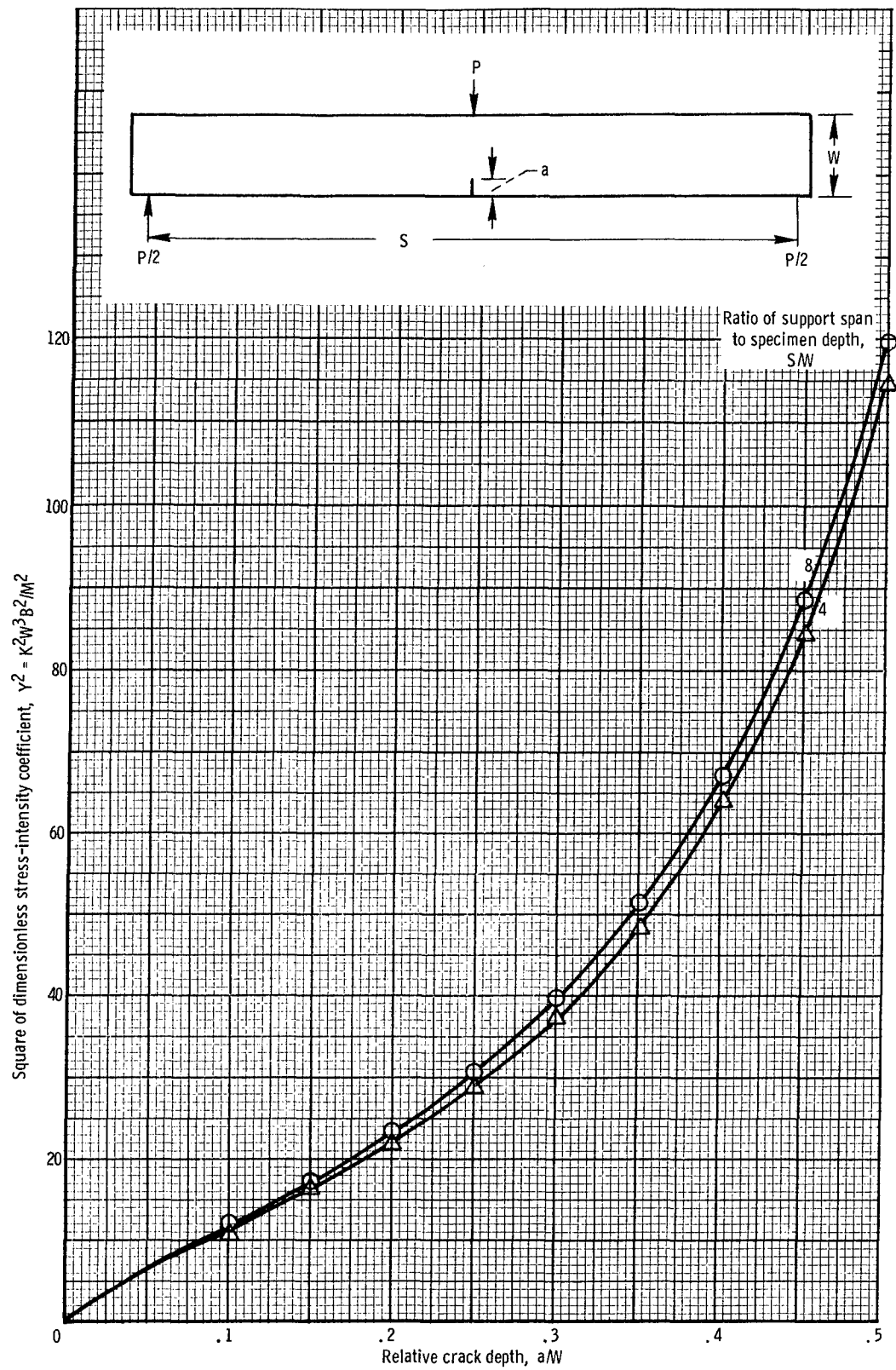


Figure 3. - Dependence of square of stress-intensity coefficient for three-point bending on relative crack depth. Bending moment, $PS/4$.

necessary to use bend specimens having cracks deeper than about 0.35 W. The results are fitted by equations (4) and (5) with an average deviation of less than 0.2 percent, the maximum deviation not exceeding 0.5 percent. The corresponding equation for pure bending (ref. 4) is

$$Y_{PB}^2 = 139 a/W - 221(a/W)^2 + 783(a/W)^3 \quad (6)$$

Comparison With Results of Independent Studies

The results of the present study for S/W equal to 8 are compared with the results of references 5, 6, and 7 in table I (p. 3). In general, the present results are in between the experimentally derived results of references 5 and 6, those of reference 5 being lower than the collocation values and those of reference 6 being higher. The results from reference 6 in table I are calculated from the fitting equation to the experimental data that is given in that reference. Values for a/W greater than 0.35 are omitted because the fitting equation obviously does not represent the experimental results in this range.

While the present results and those of reference 6 agree fairly well, the agreement is not as good as that demonstrated in reference 4 between the boundary collocation results for pure bending and experimental results for four-point bending. Furthermore, the agreement between the two sets of experimental results for three-point bending (refs. 5 and 6) is distinctly poorer than the agreement of either with the present boundary collocation results. These facts would appear to indicate that it is inherently more difficult to obtain an accurate K calibration for three-point bending than it is for four-point bending.

Reference 7 cites only three actual results from Bueckner's unpublished analysis of notched bars in bending. These results, converted to values of Y^2 , are listed in table I. For a/W equal to 0.2, the agreement with the present result is quite good, in fact, better than the agreement with either set of experimentally derived results. The agreement for a/W values of 0.1 and 0.5, however, is no more than fair. This is a matter of academic rather than practical concern since the optimum range of a/W for K_{Ic} testing with bend specimens is between 0.15 and 0.35.

K_{Ic} Measurement Capacity in Relation to W

As discussed at some length in reference 1, the greatest value of K_{Ic} that can be measured accurately with a given bend specimen, or the K_{Ic} measurement capacity of the specimen, depends on both the depth W and the thickness B of the specimen. The

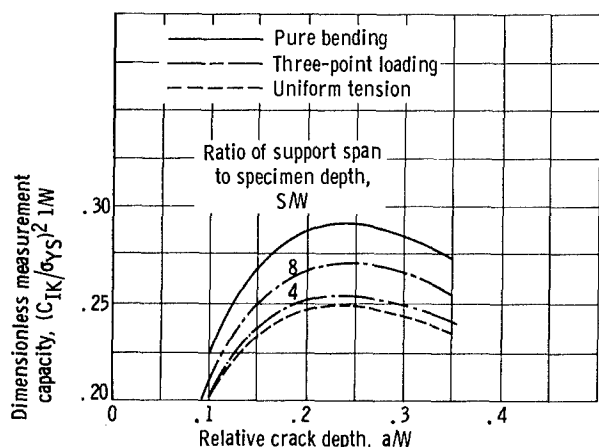


Figure 4. - Dependence of plane strain crack toughness measurement capacity on relative crack depth for single-edge-notch specimen subjected to various types of loading.

dependence on B is unrelated to the present work and will not be discussed here. Given a bend specimen of adequate thickness, the symbol C_{IK} is used to denote the maximum value of K_{Ic} that can be measured with acceptable accuracy by using that specimen. The currently accepted criterion for evaluating C_{IK} is that the nominal stress at the position of the crack tip should not exceed the yield strength of the material in a valid K_{Ic} test; that is, $6M/B(W - a)^2$ should not exceed σ_{YS} . Substituting in the expression

$$Y^2 = K^2 B^2 W^3 / M^2, \text{ and transposing give}$$

$$C_{IK}^2 / \sigma_{YS}^2 W = Y^2 (1 - a/W)^4 / 36 \quad (7)$$

The dimensionless quantity $C_{IK}^2 / \sigma_{YS}^2 W$ is a K_{Ic} measurement efficiency factor which can be similarly evaluated for other types of specimens, as in reference 1. In all cases it is a function of a/W since Y^2 is a function of a/W . In the special case of three-point bending it is also somewhat dependent on S/W , to the same extent that Y^2 depends upon S/W . For any given values of a/W and S/W , there is a unique value of $C_{IK}^2 / \sigma_{YS}^2 W$, and therefore, C_{IK} is proportional to the yield strength of the material and to the square root of the specimen depth W .

Plots of $C_{IK}^2 / \sigma_{YS}^2 W$ against a/W for three-point bending with S/W equal to 4 and 8 are shown in figure 4. Also shown for comparison are similar plots for pure bending and uniform tension, taken from references 4 and 3, respectively. The measurement efficiency is greatest when a/W is about 0.25 in all cases. Furthermore, over the range of a/W from 0.15 to 0.35 the measurement efficiency is within 10 percent of the maximum value in each case. The differences in K_{Ic} measurement capacity according to the different methods of loading shown in figure 4 are insufficient to be of much practical importance. The degree of uncertainty about the criterion used to calculate the measurement efficiency factors precludes drawing any fine distinctions in this respect. For practical purposes, therefore, it is reasonable to assume that the measurement capacity of a single-edge-notch specimen will be independent of the manner in which it is loaded. A convenient working rule for all single-edge-notch specimens is that C_{IK} is about $\sigma_{YS} W^{1/2} / 2$ (for a/W in the range 0.15 to 0.35).

CONCLUDING REMARKS

In this report the results obtained by the boundary collocation procedure are expressed in general form in terms of the square of the dimensionless stress-intensity coefficient $Y^2 = K^2 B^2 W^3 / M^2$. In the case of pure bending, Y^2 is a function of the relative crack length a/W only. In the case of three-point bending, the computed values of Y^2 were appreciably lower than the corresponding values for pure bending (computed previously by the same method), the more so the smaller the ratio of support-span to specimen depth S/W . Thus, a different K calibration plot of Y^2 against a/W is needed for each different value of S/W that is used in plane-strain crack toughness testing with three-point bend specimens. Accurate plots of Y^2 against a/W and fitting equations are given for S/W equal to 8 and 4.

For S/W equal to 8 the maximum deviation of Y^2 for three-point loading from that for pure bending was about 7 percent. For S/W equal to 4 the maximum deviation was about 13 percent. Since Y^2 is proportional to K^2 , the corresponding deviations for K are approximately half as great. It is considered that results for S/W substantially less than 4 would be of dubious accuracy and probably more misleading than useful.

The K calibration relation of Y^2 to a/W is also slightly affected by the spread of the contact pressure region around the center loading point. This spread depends upon the yield strength and toughness of the material and the size of the specimen tested. Over the practical range for acceptable plane-strain crack toughness measurements, the effect of this factor was at most 1 percent and can therefore be considered negligible.

The results agree fairly well with published results derived from experimental compliance measurements; one set gave higher values of Y^2 than the present method, and the other set gave lower values.

The plane-strain crack toughness measurement capacity as related to the depth of three-point bend specimens was estimated to be somewhat lower than that of four-point bend specimens, but for practical purposes this difference is probably of little importance. For all single-edge-notch specimens, whether tested in tension or bending, the measurement capacity is greatest in the range of a/W between 0.15 and 0.35.

Lewis Research Center,

National Aeronautics and Space Administration,
Cleveland, Ohio, August 20, 1965.

APPENDIX A

SYMBOLS

a	crack depth	r	polar coordinate referred to crack tip
B	specimen thickness	S	span
C_{IK}	estimated maximum value of K_{Ic} that can be measured with specimen of given dimensions and yield strength	V	distance from crack to boundary selected for collocation analysis
d_{2n}, d_{2n-1}	coefficients of Williams stress functions	W	specimen depth
K	stress-intensity factor of elastic-stress field in vicinity of crack tip	x, y	Cartesian coordinates referred to crack tip
K_{Ic}	critical value of K , at point of instability of crack extension in first or open mode, a measure of plane-strain crack toughness of material	Y	dimensionless stress-intensity coefficient, $KBW^{3/2}/M$
M	applied bending moment	$2y_0$	length over which applied load at center was assumed to be distributed
m	number of selected boundary stations used in collocation computation	θ	polar coordinate referred to crack tip
P	total load applied to specimen	σ_x	stress component in x-direction
		σ_y	stress component in y-direction
		σ_{YS}	0.2 percent offset tensile yield strength
		τ_{xy}	shearing stress component
		χ	stress function

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